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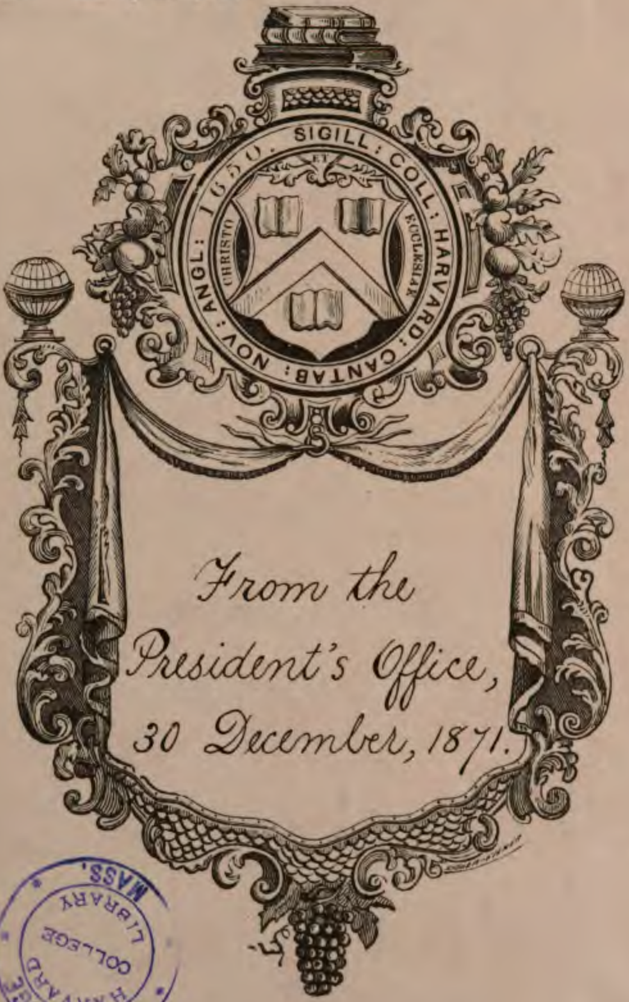
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THE PRINCIPLES

OF

PURE CRYSTALLOGRAPHY;

A LECTURE-GUIDE,

BY

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PREFACE.

These few pages are intended both for students and for investigators.

To my *students* these pages will constitute a guide to the course of lectures on pure crystallography, delivered each fall term (September to December), at the Chemical Laboratory of the State University of Iowa. For their use it may be bound up with enough blank paper to enter diagrams and notes taken at the lectures, so that they have the whole subject in one volume.

To the *investigator* the author offers these pages as an excuse for the English edition of Atomechanics promised two years ago, particularly since demands for that work have recently become more frequent again, especially from Germany. The reader will, in chapters V. and VI., find a concise exposition of the present status of Atomechanics in relation to crystallography.

Our treatise on the so-called crystal-systems, especially the quadratic, hexagonal, and tesseral forms, is herein offered to the careful consideration of all crystallographers. We have united the opposing views of *Schrauf* and the older crystallographers, and trust that, also, here the mean may approach truth more than either of the separate determinations. See articles 48 to 60, and 80 to 100.

Our crystallographic notatation (see 11, 62, 64, 67), will perhaps be found to unite the simplicity of the notation of Miller, with the directness of the notation of Weiss. Why we cannot use Miller's may be seen from 147 to 150.

Some years ago we sought recreation in the study of Chinese. In the fourth book of *Lun-Gnee*, we found (I. 8), a few words which the younger readers of this synopsis may often have occasion to think of while studying these pages. We therefore quote *Chee:* "Who do not strive to learn, to them I do not unfold my Ideas; who open not their minds, those I do not labor to instruct. When I describe one corner, if the pupil comprehend not the other three, I do not repeat my instructions."

CONTENTS.

CHAPTER I. Crystal Elements 1
II. Crystal Drawings 7
III. Crystal Forms 11
IV. Crystal Descriptions 20
V. Crystal Symmetry 30
IV. Crystal Atoms 41

CRYSTALLOGRAPHY.

CHAPTER I.

Crystal Elements.

1. A CRYSTAL *is a polyhedron formed spontaneously by its own particles.*

CRYSTALLOGRAPHY *is that branch of physical science which treats of the form of crystals.*

These definitions are deduced from observations on crysallization and crystals in HINRICHS' ELEMENTS OF PHYSICS* Chapter III., Section V. and VI, where also the other necessary definitions here used occur. Only after having carefully worked his way through these two sections—or their equivalent—can the student successfully master the present very concise treatise of pure crystallography. At the same time it is essential that the student should be *practically* familiar with Section VI. of Chapter I. of the same work, and especially with Articles 71 to 80, treating of co-ordinates in the plane and in space.

2. Any two *zones* of a crystal will determine three *axes*, X, Y, Z, (Physics, 199). The three *axial-planes*, X

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*For the sake of brevity we shall refer to the above work as PHYSICS and give the number of the article referred to. Thus Physics, 199, means article 199 in the above work.

Y, Y Z, Z X need only be laid parallel to three different crystal faces, such as P, M, T, (Physics, 168, 169). In this case X would be the axis of zone P M, Y of zone P T, and Z of zone M T. Faces P, M, T, are called *pinacoids*, if they are parallel to the axial planes.

3. The crystal is usually placed in such a position that the *third* axis Z is *vertical* (Physics, 80). The *first* or *proto-axis* X is directed from the center O towards the right of the observer.

4. The magnitude of the *inter-axial angles* evidently depends upon the crystal described (Physics, 173,) and upon the faces selected on this crystal. (2.) However, the faces are usually selected so as to *make the axial angles as nearly right angles as possible.*

These inter-axial angles are denoted by letter α, β, γ, namely the rotation around by

X	Y	Z
α – y z;	β – x z;	γ – xy.

5. Accordingly we have the following four possible cases:

1°. *None* of these angles is exactly 90°; or $\alpha \gtrless \beta \gtrless \gamma \gtrless 90°$. Such crystals are termed *triclinic.*

2°. Only *one* of the interaxial angles is exactly 90°; $\alpha \gtrless \beta \gtrless \gamma = 90°$. Such crystals are termed *diclinic.*

3°. Two of the interaxial angles are exactly 90.° $\alpha \gtrless \beta = \gamma = 90°$. Such crystals are termed *monoclinic.*.

4°. Finally *all three* of the interfacial angles may be exactly 90°; that is, $\alpha = \beta = \gamma = 90°$. Such crystals we term *orthoclinic.*

6. Besides the three pinacoids used for the determination of the axial planes (see 2) a crystal has at least one more face, F, intersecting the above three faces, and therefor also intersecting the three axes. The lengths cut off on the axes by this face F we term the *fundamental axes* of the crystal, and denote them by the letters a, b, c. The face F is also called the *fundamental face* of the crystal.

7. If the crystal possesses many faces besides the three pinacoids, any of the three faces *may* be selected as the fundamental; but we shall see that in many cases some one of all three faces is distinguished from all others of the same crystal by certain important properties, which show that face to be *the* fundamental one of the crystal.

8. The *magnitude* of the three angles α, β, and , and the *length* of the three fundamental axes a, b, c, constitute the six ELEMENTS *of a crystal.* Since, however, the angles only of crystals are of importance, and not the absolute linear dimensions, (*Steno's Law*, Physics, 173,) we can always put one of the fundamental axes equal to unity. We select usually $b = 1$.

9. Any other face f of a crystal will be completely determined by the three *parameters* x, y, z, cut off by the face from the three axes X, Y, Z

10. Observation has proved that the parameters x, y, z of any crystal face f stand in a simple, *rational* ratio to the fundamental axes a, b, c; that is, to the parameters of the fundamental face F. Consequently:

$$\frac{x}{a} : \frac{y}{b} : \frac{z}{c} = h : k : l.$$

The letters h, k, l are accordingly *simple*, *entire numbers*, and we may put the parameters

$$x = h. a. \qquad y = k. b. \qquad z = l. c.$$

11. The simple, whole numbers h, k, l, are termed the *indices* of the face f, and when written one after another *in the order* hkl, constitute the *formula* of the *face* f. The *first* index corresponds always to X, the *second* to Y, the *third* to Z. Thus 235 is the formula of a face intersecting the first axis (X) at a distance 2 a, the second axis (Y) at a distance 3 b, and the third axis (Z) at a distance 5 c. The *parameters* of this face are 2a, 3b, and 5c; the indices are 2, 3 and 5.

Hence, also, the formula of the fundamental F is 111.

12. If the face f intersects any of the negative axes, the corresponding parameter, and therefore also the corresponding index becomes negative. For the sake of brevity the *minus* sign is written *above* and not before the index. Thus

$\bar{2}35$ is a face, which intersects the X axis at the distance —2a or 2a to the *left* of the origin O. (Physics, 80 and 71.)

13. *The law of simple rational indices* (10) distinguishes a crystal from any artificial polyhedron. On some crystals many hundred of separate faces have been observed, (compare Chapter IV.;) but they are all *accurately* represented by simple whole numbers h k l. The most frequent values of these indices are 1, 2, 3; also infinity, if the face be parallel to some axis.

14. A face hkl intersects all three axes X, Y, Z, unless one or two of the indices is infinite. Faces intersecting all three axes are termed *pyramidal* faces. Thus hkl 2 3 5, 1 2 1 are pyramidal faces.

If, however, the face is parallel to an axis it is called a *prismatic face*, and the corresponding index is infinity. Thus hk∞, h∞l, ∞kl are prismatic faces. The last two faces, parallel to the not vertical X or Y, are called *domatic* faces.

If, finally, the face is parallel to two axes, it has both the corresponding indices infinite, and is termed a *pinacoid.* Thus 1∞∞ is the *deutero-pinacoid*, parallel to the axial-plane Y Z; ∞1∞ is the *proto-pinacoid*, parallel to the axial-plane X Z, and ∞∞1 is the *basal pinacoid* parallel to X Y.

15. *A simple form is composed of all those faces f which possess the same numerical indices h k l, although of different sign, provided the faces have the same inclination to the three axes.* The *formula* of such a simple form is (hkl) the bracket (— — —) indicating that *all possible changes in sign* are to be made, consistent with the preservation of the inclination to the axes.

16. Crystal forms, composed of more than the faces of one simple form, are termed *combinations* of the several simple forms composing the same. Thus a crystal bounded by the forms (123) (111) (157) (132) (113) etc., would be a *combination* of these forms.

17. In the description of such combinations it should always be stated which simple form or forms *dominate.* This, together with the formula of the component forms constitutes the *habitus* of the crystal (Physics, 172).

18. Often crystal faces exhibit certain peculiarities; but usually *all the faces of the same simple form possess the same peculiarities.* Such facts are intimately connected with the formal habitus, and should be mentioned in connection therewith.

19. *An interfacial angle* is denoted by writing the symbols of the two faces in succession. Thus $235 : 2\bar{3}5$ is the interfacial angle between face 235 and face $2\bar{3}5$.

Accurate measurements of such interfacial angles should be recorded with the description of the crystal.

20. It is easily seen that the interfacial angles are determined theoretically by the *elements* of the crystal and the *indices* of the faces; but the values of interfacial angles thus determined are distinguised as *calculated* from those obtained by direct measurement.

21. These calculations require the use of spherical trigonometry. It is, however, quite sufficient for beginners to determine some of the most important angles by *construction* from the elements, by the methods of linear drawing, given in Physics, 51—80. It is especially important and very easy to construct the angles in the three axial zones, in orthoclinic crystals.

22. The *elements of a crystal* are obtained from the observed angles by like method of calculation or construction. Thus in orthoclinic crystals the elements are determined by means of two zones, such as the vertical parallel to the axis Z, and the horizontal parallel to the axis Y or to X. In potassium nitrate (Physics, 181) the zone Mb gives, by the angle MM' the ratio a : b, while the zone Dd b will, by the angle b D, give the ratio a : c, so a : b : c are determined. Besides $\alpha=\beta=\gamma=90.^\circ$ Practice!

23. Having determined the elements, it remains to determine the *indices* for the observed faces. This may again be accomplished by calculation or, in the simple cases, by construction. Thus, in the above case, we have evidently determined the elements by the faces M and D of nitre, so that M is 1 1 ∞ and D is 1 ∞ 1. If in the zone Ddb also face d is drawn, according to its observed inclination (Physics, 181) it will be found that d is 1 ∞ 2. Also that b is 1 ∞ ∞.

The crystal descriptions in *Physics* 182, 183, 184 may be used in the same manner for the determination of the elements of these crystals.

24. *Four faces are sufficient to determine the elements of any crystal.* Three faces will determine the inter-axial angles α, β, γ, (see 2 and 4;) while the fourth face may be taken as the fundamental F (see 6). In order that this latter can be fixed in position towards the former three the inclinations of F with any two of the former are required. Hence the total number of *angles* sufficient to determine the elements of a crystal is *five.*

25. But it is well to take more than this least possible number of angles into account, in order, as far as possible to determine the unavoidable *errors of observation* and the slight *irregularities* which crystals at times exhibit.

The best method consists in calculating the elements from any five observed angles. Then by comparison with the observed angles the *errors* can be obtained.

Now change the elements to a slight extent, and repeat all calculations. A new set of errors will thus be obtained.

Since very small changes in any two variables always are nearly proportional, data have now been obtained for the determination of *that* change in the elements which would reduce all the errors, as nearly as possible to zero. (*Von Lang*, Krystallographie, Wien, 1866, p. 351—352.) For applications see *Von Zepharowich*, Krystallographische Studien über den Idokras, Wiener Sitzungsber. 1864, Bd. 49. Abthl. I., p. 21—24.

The method of least squares has been used for this purpose, in a less elegant manner, by *Schabus* 1855; also, for the angles of pyroxene, by *von Kokscharow*, Materialien zur Mineralogie Russlands, Bd. IV., pp. 370—385.

CHAPTER II.

Crystal Drawings.

26. The common *Crystal drawings* are either horizontal, vertical or oblique projections of the crystal. See Physics, 75. A very good representation is obtained by the common oblique projection together with a horizontal projection of the same crystal. In this manner crystal forms are figured in the excellent *Atlas* accompanying *Von Kokscharow's* Materialien zur Mineralogie Russlands.

27. The most common *oblique* projection used exhibits the crystal's front turned about 18° to the observer's left, and the crystal's top turned about 6° down towards the observer. The first angle is termed the angle of declination, the last the angle of elevation.

Otherwise the oblique projection is *axonometric;* the rate being very nearly along X and Z full scale, but along Y *one-third* full scale. See Physics, 76. First the elements are constructed, then the faces are entered by their indices. Practice!

28. Besides these crystal drawings two indirect methods of representing crystal forms on paper have proved very useful; they are the so-called *methods of projection of Quenstedt* and *Miller*, which, however, are both developements of a method first proposed by *Neumann.* Large projections of this kind (for h one or several dm) I find very useful in the lecture room.

29. *Quenstedt's Projection is a linear projection.* It is obtained on plane a b by intersection of *projection planes* laid through the extremity of the axis c, and parallel to the crystal faces themselves.

30. Accordingly *each face will be represented by a line* called the *trace* of the face. Only the basal face cannot be represented in this projection, because it does not intersect the plane a b. The trace of any face parallel to the vertical axis c passes of course through the point of intersection O of the axes a and b; that is the faces h k ∞ have traces passing through O, and parallel to a line cutting off the indices h and k on a and b.

All other faces have traces of the indices μ and ν or a and b in the projection, determined by

$$\mu = \frac{h}{l} \qquad \nu = \frac{k}{l}$$

31. Since all the edges of any one zone are mutually parallel, these edges are all projected in the same point, which is called the *zone point. The traces of all the individual faces of the zone pass through this zone-point;* for the projection plane of each face also contains the common projection-line of the edges.

The point of intersection O of the axes a and b is thus the zone point of the zone parallel to axis c. See 30.

32. It is easy by direct observation to ascertain whether any given face f belongs to a given zone; for we need only to rotate the crystal around the axis of the zone so that the light is successively reflected from the different faces of the zone to our eye—if the face f also in its turn reflects the light to our eye, it is in the same zone with the other faces in question.

33. If now a face f is thus found to belong to *two dif-*

ferent zones, the zone-points p and p′ which are known in the projection, we can draw the trace of the face f in the projection; for it is the straight line passing through the points p and p′. Hence also the indices μ and ν will be determined, and therefore also the full formula h k l of the face (see 30.) This is the *deduction* of the face.

34. Thus the mere observation of the *zonal relations* of a face very often suffices to accurately determine its formula without any calculations whatever. Herein consists the principal practical importance of *Quenstedt's* projection. Practice the same on some of the crystals studied. Especially Physics, 181 to 183.

35. If carried out in the usual way, by full lines, the projection becomes often an intricate net-work of lines. I therefore draw the trace full only *between* the two axes it intersects, or near the centre of the figure, and *faintly dot* the continuation of the trace. In this manner the projection also brings out the symmetry of the crystals, and becomes really elegant.

36. I also add the vertical projection of the extremity of c, through which all projecting planes are laid, and somewhere near the margin of the drawing mark the *length* of that axis c (to scale.) Only in this way is it easy to obtain at any time immediately a *correct* view of the crystal form, especially in case of triclinic and monoclinic forms.

37. In *Miller's Stereographic Projection* a sphere is described around the center of the crystal, and each face F moved parallel to itself until it touches the surface of this sphere; this point is called the *normal point*, n, because the radius to this point is normal to the crystal face.

The plane of projection (the paper) is usually passed through the axial plane a b; at times, however, at right angles to c. This plane of projection will intersect the sphere in a circle which is drawn on the paper and termed the *circle of projection*. From the center of this circle a perpendicular is drawn down until it intersects the spherical surface below in a point termed the *nadir*, N.

The nadir N is joined by a straight line with the normal point n of the face F. The point p where the line of projection Nn intersects the place of projection, (the paper) constitutes the *projection* of the normal point, and thus the projection of the face F.

38. It is evident, that each face F will thus be accurately determined in position by its projection p; and also that the entire projection will fall in, or inside, the circumference of the circle of projection.

It is also readily seen, that the zone at right angles to the plane of projection is projected in the circle of projection.

Since the normal points n of any zone are in a plane passing through the center of the sphere, they are in a great circle on the sphere; and therefore will also form an arc of a circle in the projection. Hence it is also quite easy in this projection to trace up the faces belonging to any zone.

39. The angle between the radii of the sphere drawn to different normal points n, projected in the points p, can easily be determined by very simple construction, at least if the points p are in a known zone. But this angle is evidently the angle between the normals of the two faces, or the *inter-normal angle*, n.

40. It is easily seen that the inter-normal angle is the supplement of the *interfacial angle* f; or n+f—180°. Hence we may, according to circumstances, either determine the internormal or the interfacial angle between any two faces in the crystal. By means of *Haidinger's* method (Physics, 171) both of these angles are observed with equal facility. By the application goniometer we obtain the interfacial (see Physics, 170). But by the more accurate *reflecting goniometers** we read off the internormal angle direct; hence these are of late more frequently given in crystal descriptions, than the interfacial angles.

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*The reflecting goniometer of von Zepharowich in Prague enables him to determine the angles to within five seconds exact! See Wiener Berichte, 1861; Bd. 43, Abth. 2, p. 547.

For a crystal face 2 mm long this is within 0.00005 mm of linear extension.

Always be careful to ascertain which of these angles are referred to in crystal descriptions!

◆

CHAPTER III.

Crystal Forms.

41. THE SYSTEMATIC DESCRIPTION OF A CRYSTAL may now be understood. Such descriptions are best given in the following order.

1. *The elements.* See 8 and 22.

2. *The simple forms observed.* See 15. Giving the *letter* used to denote the faces, and also the formula of these faces. In this enumeration it is not always necessary to write the brackets.

In this enumeration it is essential to distinguish between the full and partial forms, more completely characterized below as *holohedral* and *hemihedral* forms. *Twin* forms should also be enumerated. See 61.

3. The enumeration of observed forms should be accompanied by a *projection* representing all the faces observed, either according to *Miller* (37) or by our modified form (35 and 36) of *Quenstedt's* projection (29).

4. The *habitus* (Physics, 172) of the crystals should next be fully given, as far as possible in reference to methods of crystallization for artificial, and to locality and occurrence for native crystals. Also the *peculiarities of the faces* of simple forms (18) should be described, such as their *striations*, etc. Compare 17.

5. In this connection some of the most important of the *physical properties* of the crystal, varying with the faces, should be given, especially the *cleavage*. See Physics, 197).

6. The *calculated* values of the interfacial or internormal angles should be tabulated, and

7. These *calculated angles should be critically compared with the observed values*, thereby finally leading to

8. A discussion of the *influence of composition and external circumstances on the elements* or the crystal form. It will then appear, that the elements α, β, γ and a, b, c, are *not* constants, but *functious* of atomic weight, temperature, and other variables.

42. A crystal-form having all faces in parallel pairs is *holohedral* (Physics, 169). If no two faces on a form are parallel the form is *hemihedral.*

For any face hkl the holohedral form therefore also contains $\bar{h}\bar{k}\bar{l}$ while the hemihedral forms do not contain the face $\bar{h}\bar{k}\bar{l}$.

43. If all faces possible according to the degree of symmetry of the elements are present in a simple holohedral form, the latter is *holosymmetric;* if only half the number are present the form is *hemisymmetric.*

The same terms are used for hemihedral forms.

44. In a *triclinic crystal* (5) *the simple form (hkl) consists of the two parallel faces only.* For a change in sign in any of these indices produces a change in inclination to the axes in the corresponding face. Consequently no really hemihedral forms are possible in a triclinic crystal.

The *diclinic* forms agree in number of faces with the triclinic crystals.

45 *The monoclinic crystal* (5) has simple forms composed of *four* faces. For hkl and $\bar{h}$kl are equally inclined to all axes, because X is at right angles to both Y and Z. They form an inclined zone. The two faces parallel to these increase the number to four in the simple form of monoclinic crystals.

46. *In the orthoclinic crystals* the four faces p=hkl, q=$\bar{h}$kl, r=$\bar{h}\bar{k}$l, s=h$\bar{k}$l have the same inclination to the three axes; they form a *four sided rhombic pyramid* around the *upper* half (l positive) of the vertical axis.

Their parallel faces p′ q′ r′ s′ (1 negative) give an equal rhombic pyramid below, so that the *holohedral* simple form (h k l) of an orthoclinic crystal is a double four-sided rhombic pyramid; we shall, for the sake of brevity, term it an *octaid*, because it is bounded by *eight* equal triangles.

47. If the axes of the octaid are all different, then the octaid is the holohedral form. Such orthoclinic crystals are termed *rhombic*, because the three axial sections are rhombs.

48. If, however, two axes are exactly equal, then each octaid (hkl) must occur in two positions, of which one results from the other by a rotation of 90° around the third axis. Such crystals are termed *quadratic.*

The holohedral simple *quadratic* form is a double eight sided pyramid of 16 faces, called *dioctaid*, and represented by the formula

$$[hkl]=(hkl)\ (0^\circ+90^\circ)$$

where the octaid (hkl) is indicated to be taken for Y in the direction of axis b and *also* for Y in the direction of axis $\bar{a}$, at 90° from the former The brace [ ] is used here and in the subsequent to indicate the combination of the orthoclinic octaid (46) in the different positions determined by the degree of *rotary symmetry* peculiar to the crystal. The vertical axis c is the axis of rotary symmetry, also termed the principal axis.

49. If the orthoclinic octaid (46) has the axes $a : b = \sqrt{3} : 1 = 1.73205 : 1$, the axial section *ab* is a rhombus of *exactly* 120°, which implies the *regular hexagon.* In this we have three equal diameters, mutually distant by 120°.

Hence the octaid in this case must be repeated 3 times, with the axis Y at 0°, at 120° and at 240°. Consequently the holohedral simple hexagonal form is a *tri-octaid*, or a double twelve-sided pyramid of 24 faces, called the *dihexagonal pyramid.* It is represented by

$$[hkl] = (hkl)\,(0^\circ+120^\circ+240^\circ).$$

50. If in a quadratic crystal (48) the third axis c becomes equal to the axis a=b, then also a and b become axes of the same degree of rotary symmetry, which c possesses in the quadratic crystals. Hence we have for *each* axis a, b, c such a diquadratic pyramid, and the holohedral simple form has 48, or three times the number 16 of faces which it has in the quadratic crystal. The formula of this form is evidently

$$[h\ k\ l] = \left\{\begin{array}{l}(hkl)\ (0^\circ+90^\circ)\\ +\ (hlk)\ (0^\circ+90^\circ)\\ +\ (lhk)\ (0^\circ+90^\circ)\end{array}\right.$$

if l is the index on the axis around which the rotation of 90° takes place. This form is a *triakis dioctaid*, but it is usually called the *Hexakisoctahedron*, because it is bounded by six-times-eight or 48 faces. *Haidinger* has called it the *Adamantoid*, because it has been observed on the diamond.

Crystals, wherein a—b—c are termed *quadrotesseral*, because for $h=k=\infty$ the simple from [hkl] becomes $[\infty\ \infty\ 1]$ which is the cube or tessera, also called *hexahedron*.

51. In the hexagonal crystal (49) the vertical axis c may, of course, have any value. But if $c=\sqrt{2}=1.41421$ the hexagonal form $[\infty\ 11]$ consists of two cubes* $^{\pi}[\infty\ 11]$, and $^{\pi}\ \infty\ \bar{1}1]$ turned 180° against one another around a diagonal as the vertical axis of rotation. These forms we term *rhombo-tesseral.*

52. Mathematically the quadro-tesseral forms are identical with the rhombo-tesseral forms, but changed in position. Hence they are, by probably all crystallographers, referred to as *tesseral* forms without distinction.

We shall soon see that these forms are physically, and therefore crystallographically, very different from one another. Compare 99, 100.

53. We have now learned to distinguish the following six crystal forms, the elements of which successively specify to simpler values, while at the same time the holohedral form attains to a constantly greater number of faces expressive of the increasing *degree of symmetry.*

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*See 84 for the signification of π.

I. CLINIC CRYSTALS; *the interaxial angles not all right angles.*

1. *Triclinic crystals;* no interaxial angle a right angle. (hkl) a pair of *two* faces.

2. *Monoclinic crystals;* two interaxial angles are right angles; (hkl) a dome of *four* faces.

II. ORTHOCLINIC CRYSTALS; *the interaxial angles are all right angles.* $\alpha=\beta=\gamma=90°$.

3. *No rotary symmetry;* $a \lessgtr b \gtrless c$. *Rhombic* forms (hkl) an octaid of *eight* faces.

4. *Rotary Symmetry of* 90° *around c;* $a = b \lessgtr c$. The *quadratic form* [hkl] is a dioctaid of 16 faces.

5. *Rotary Symmetry of* 90° *around each axis, a, b and c;* $a=b=c$. The *quadro-tesseral* form is a *triakisdioctaid* or *hexakisoctahedron* of 3×16 or 6×8, that is forty-eight faces.

6. *Rotary Symmetry of* 120° *around c;* $a=b\sqrt{3} \lessgtr c$. The *hexagonal* form is *tri-octaid* or a double twelve-sided pyramid of $3\times8 = 2\times12 =$ *twenty-four* faces.

7. Rotary Symmetry of 120° around one axis, combined with rotary symmetry of 90° around another axis. The *rhombo-tesseral form*, geometrically the same as 5, but different in position and in structure.

54. All crystals of one kind of form and possessing the same degree of symmetry constitute a *crystal system* of the same name as the form. Thus all triclinic forms constitute the triclinic system, etc.

55. When the forms of different crystals are compared the elements of these crystals must be considered as *variables* (see 41, 8). The forms of higher symmetry thus arise through *special values* of these variables, so that the *systems pass gradually into one another.* Instead of forming distinct *classes* of crystals, the forms of higher symmetry are only limits or transition-values between those of other degrees of symmetry.

Thus the *quadratic forms* constitute a transition or limit between the rhombic crystals wherein $a=b-$ and those wherein $a=b+\epsilon$, if ϵ is a variable positive quantity.

56. The elements of all orthoclinic crystals may be exactly represented by a *point* in a co-ordinate-plane, if a, the axis of X (horizontal) and c, the axis of Z (vertical), provided $y=1$ be understood. Then the ordinate at $x=1$ is the place of all quadratic forms, and on this line $c=1$ is the place of all quadro-tesseral, while above this point we have *acute* quadratic forms, below it we have *obtuse* quadratic forms. So also the ordinate as $x=\sqrt{3}$ represents all hexagonal forms, while in this ordinate the point $c=\sqrt{2}$ represents all rhombo-tesseral forms. The balance of the plane represents the rhombic crystals proper.

See my *Atomechanik*, 1867, p. 28—29.

57. The otherwise intricate study of the different holohedral and hemihedral forms in the crystals of higher degree of symmetry (48, 49, 50 and 51) becomes quite simple by means of the following analysis of the forms in rhombic crystals. For the sake of brevity we shall denote the eight faces of the holohedral rhombic pyramid by the letters p q r s (upper faces) and p′ q′ r′ s′ (lower parallel faces) as explained in 46.

To distinguish the holosymmetric holohedral form from the others which are either not holosymmetric or not holohedral (see 42, 43) we shall denote the full octaid by ω (h k l.)

58. The true hemihedral rhombic form is the *sphenoid*; the *right* σ [hkl] formed by the four faces p q′ r s′, and the left σ $[\bar{h}kl]$ formed by p′ q r′ s.

59. We also distinguish the following *monozonal forms :*

Clinozonal, π (h k l) of 4 faces p q p′ q′.

Orthozonal, δ (hkl) of 4 faces p r p′ r′.
Also the *monopyramidal forms :*
Orthomonopyramid, ζ (h k l) of 4 faces p q r s.
Deuteromonopyramid ζ (h k l) " " " p q r′ s′.
Protomonopyramid ξ (h k l) " " " p′ q′ r s.

Of each of these forms there are two differing in the sign of one index.

60. By two of these letters before the indices of a form we denote those faces, which both separately contain in common. The most important cases are:

$\delta\,\zeta$ (h k l) composed of p and r′.
$\delta\,\sigma$ (h k l) " " p " r.
$\xi\,\sigma$ (h k l) " " p " q′.
$\zeta\,\pi$ (h k l) " " p " q.

Of each of these forms there are *four* differing in position.

61. *Twin-Crystals* are aggregates of two (or more) crystals united after some simple mathematical law, but having not all their axes and faces parallel. Taking any one of these individual crystals as firm, the position of the other individual is obtained by a rotation of 180° around some definite real crystal-line (axis, edge or normal to a face) in the first individual.

62. Before we can proceed to the special description of the crystal forms observed, we must explain the *principal systems of notation* in use, which differ more or less from our own.

63. *Miller's* notation (p q r) is the inverse of our own; for it gives the parameters as

$$\frac{a}{p} \quad \frac{b}{q} \quad \text{and} \quad \frac{c}{r}$$

Hence the face p q r Miller is identic with h k l Hinrichs, if

$$h : k : l = \frac{1}{p} : \frac{1}{q} : \frac{1}{r}$$

for all except the hexagonal forms. Miller's axes for the latter are triclinic $\alpha\ \beta\ \gamma \gtrless 90°$, and admit therefore of no very simple transformation to our own. By means of *Schrauf's* orthohexagonal symbols the transition from Miller's to our own symbols can, however, be readily effected.

64. The notation of *Weiss* is ha : kb : lc, without any restriction as to the order of these parameters.*

Our own notation results from this by adopting the *fixed* order of the succession of the parameters, (see 11) and thus becoming enabled to drop the letters a, b, c.

65. The notation of *Naumann* is far too symbolical and intricate to be of easy application. While a decided improvement of that of *Mohs*, it cannot be compared to those notations which give a concise and analytical expression of the crystal-face (Weiss, Hinrichs,) or crystal-normal (Miller).

66. *Levy's* notation is used by only very few French crystallographers. It is a hybrid between the notation of *Hauy*—which was very excellent in the earliest stages of crystallography—and the notation of *Weiss*. This system of notation would long ago have been abandoned if the boundless national vanity of the French did not stamp the recognition of foreign merit an act of treason.

67. To substantiate our statements we give the different notations for a simple adamantoid, see 50:

Miller........................[623.]
Hinrichs......................[123.]
Weiss.........................a : 3a : 2a.
Naumann.......................2O3.
Levy..........................$b\frac{1}{3}\ b\frac{1}{2}\ b\frac{1}{3}$.

68. We are now prepared to understand the systematic description of the observed crystal forms. We shall for each system always first give a synopsis of the forms

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*See the crystallagraphic works of *Rammelsberg* and the excellent monographies of *G. vom Rath*.

in general, that is the holohedral form (see 44 to 54) together with the specializations thereof resulting by specialization of the indices, including the hemihedral forms. In tesseral and hexagonal forms the latter will require special investigation.

69. Each system is next to be represented by some one of the excellent monographs of *Nicholas von Kokscharow*, given in his *Materialien zur Mineralogie Russlands*, published at Petersburg in five volumes of 400 pages 8vo. each, and accompanied by a most excellent Atlas of more than 70 plates in quarto. For the sake of brevity we shall refer to this work by adding the number of the volume and the page of the latter to the name of the author. Thus, v. Kokscharow, III., 139, refers to vol. 3, page 139. Also, *Pl.* 69 is to stand for plate 69 in the Atlas.

We select this work of *von Kokscharow* because in a comparatively small bulk and at a moderate cost it gives so great a number of monographies.

If, however, the SITZUNGSBERICHTE der mathematisch-naturwissenschaftlichen Classe der Kaiserlichen *Akademie der Wissenschaften*, WIEN, are accessible, the best monographies ever published on crystal species will be at hand. We mention here only the following, which have appeared since 1860:

Von Zepharowich: Idokras, 1864, 49, I., 6–134, 13 plates.
" Anglesit, 1864, 50, I., 369–374, 1 plate.
" Epidot, 1862, 45, I., 381, 1 large plate.
" Unterschw. Kalk, 1862, 45, I., 499-510, 3 plates.
Zirkel: Bournonit, 1862, 45, I., 431–466, 7 plates.
Dauber: Rothbleierz, 1860, 42, 19–54, 12 plates, especially 3 large projections.
Krenner, Antimonit, 1865, 51, I., 436–481, 9 plates.
Auerbach, Coelestin, 1869, 59, I., 436–481, 9 plates.
Brezina, Schwefel, 1869, 60, I., 1 plate.

Poggendorff's Annalen bring, at times, good crystallographic monographies, especially from *G. vom Rath.*

70. Finally we shall give a somewhat general description of some characteristic *isomorphous group*, (see Physics 183 and 194—196) *of artificial crystals for each system.*

That the study of the monographies should go hand in hand with the study of the actual crystals, both native and artificial, is simply a matter of course. It is also expected that the student constructs our *Quenstedt* Projection from the given data for each of the native crystals.

CHAPTER IV.

Crystal Descriptions*.

A. Crystals without rotary Symmetry.

I. TRICLINIC CRYSTALS.

Elements: $\alpha \lessgtr \beta \lessgtr \gamma$ 90°. a $\gtrless$ b $\lessgtr$ c.

71. The holohedral form contains (see 44) only two faces which are parallel; it is a *tetartopyramid.* According to the position of these faces we have the following simple forms in this system (see 14):

1. Pyramidal Faces: the *upper-right* (hkl); also the *lower-right*, the *upper-left* and *lower-left Tetartopyramid.*
2. Prismatic Faces: the *right* (hk∞) and the *left Hemiprism.* The *upper* (∞kl) and the lower *Protohemidoma.* The *upper* (h∞l) and the *lower Deuterohemidoma.*
3. Pinacoids: The *Basopinacoid* (∞∞1) the *Protopinacoid* (∞1∞) and the *Deuteropinacoid* (1∞∞).

Hemihedral forms are impossible in triclinic crystals.

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*We give merely a list of the examples used by ourselves in our lectures, together with a reference to the original observations which we give reduced to our notation. We also make use of the plates here referred to and carefully exhibit to our students the best demonstrative specimens accessible, both of native and artificial crystals. A *practicum* follows the lectures.

72. Anorthite. See *Von Kokscharow*, IV., 290—257, Pl. 70, 71.

Vom Rath, Pogg. Ann. 138, p. 449, p. 464.

Tschermak, Sitzungsberichte, Wien, 1864, Bd. 50, I., p. 566—613; a most important paper.

Total 35 forms with 70 distinct faces.

A very critical monogrophy is that of *G. vom Rath* on Axinite, Pogg. Ann. 1866, Bd. 128, pp. 20—16 and 227—259.

73. Blue Vitriol. See *Kupffer* in Rammelsberg, Neueste Forschungen in der Krystallograph. Chemie, p. 50. Also *Pape* in Pogg. Ann. 1868, 133, p. 364. Observed 15 forms, embracing 30 faces.

Wohlwill's isomorphous selenates of copper, iron and zinc, with the same number (5) of atoms of water.

II. MONOCLINIC CRYSTALS.

Elements: $\alpha \gtrless \beta = \gamma = 90^\circ$. $a \lessgtr b \lessgtr c$.

74. The simple holohedral monoclinic form is composed of four faces at most (see 45). According to the special values of the indices we have (see 14) the following forms:

Pyramidal: The *upper* (hkl) and the *lower* Hemipyramid; each of 4 faces.

Prismatic: The vertical *prism* (hk∞) and the *deuterodoma* (h ∞l) each composed of four faces. The latter is more commonly called the *clinodoma.* The *upper* (∞kl) and the *lower protohemidoma* is each composed of two faces only; they are often termed orthodomas.

Pinacoids: The bassopinacoid (∞∞ l) forms the *oblique base;* the protopinacoid (∞l∞) constitutes the *orthopinacoid;* and the deuteropinacoid (1∞ ∞) is useally called the *clinopinacoid.* These names have reference to X or a as horizontal (ortho-), Y or b as inclined (clino-) axes, while Z or c is vertical.

No hemihedral forms occur, but hemimorphism is possible.

75. PYROXENE. See *Von Kokscharow*, IV., 258—385 Pl. 65—69. 48 Forms with 176 faces.

76. HEXHYDRAHED HAUEROIDS:* Observations of *Brooke*, *Marignac*, *Miller*, and *Rammelsberg* in the latter's Krystall. Chemie. Also *Von Hauer's* Krystallogenetische Beobachtungen, Sitzungsberichte, 1860.

Eleven forms with 38 faces.

III. RHOMBIC CRYSTALS.

Elements: $\alpha = \beta = \gamma = 90°$. $a \gtrless b \gtrless c$.

77. The simple holohedral rhombic form is the octaid (hkl) of eight equal faces (see 46). We distinguish (see 14) the following forms:

Pyramidal: The octaid (hkl). Eight faces.

Prismatic: The vertical rhombic prism (hk∞). The protodoma (∞kl) called the *macrodome* and the deuterodoma (h∞l) called the *brachydome* if $a > b$. Each of these forms contains four faces.

Pinacoids: The basopinacoid (∞∞l), the *macropinacoid* (∞l∞) and the *brachypinacoid* (l∞∞), each of two faces.

The only closed hemihedral forms are the *positive* σ (hkl) and *negative sphenoids.* Compare 58. These sphenoids occur on the hepthydrated zinc and magnesium sulphates. See Physics, 184, 185.

Hemimorphism on a zinc silicate. Compare 59.

78. TOPAZ. *Von Kokscharow*, II., 198; II., 344; III., 195; III., 378; V., 34. The seven plates 38, a, b, c, d, e, f. Thirty-two forms embracing 184 faces.

79. POTASSIUM SULPHATE. See *Mitscherlich* in Rammelsberg, Kryst. Chemie. Nine forms embracing 38 faces.

This form is the representative of 15 sulphates and selenates, 3 chromates and manganates, 4 permanganates, 4

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*See *Hinrichs*, Sitzungsberichte, Wien, 1870, Bd. 75, Abthl. I.

perchlorates, 9 silicates, 4 formiates; in all 39 salts which I have termed *Barytoids*,* already carefully determined in my *Note sur la forme cristalline des sulfates*, Comptes Rendus, 1869, t. 68, p. 344. See also my contributions to Molecular Science, 1869, No. 3, p. 41—43.

For more complete description of this *Barytoid Form* see the monographies on Celestite, Anglesite, etc., quoted in 69. Compare also 90, the silicate *Phenacite*.

B.—Crystals with Rotary Symmetry.

IV. QUADRATIC CRYSTALS.

Elements: $\alpha = \beta = \gamma = 90^\circ$. $a = b \lesseqgtr c$.

80. The holohedral and holosymmetric simple form in this system results from two octaids (hkl) one of which having been turned a right angle around the vertical axis Z, see. 48. To make this composition more apparent it is best to draw the traces of the faces on the plane X Y. I mark the trace of an upper face by a full line, of a lower face by a dotted line, and of upper and lower face by a line crossed. Also, to distinguish the two octaids, I draw the traces of the octaid (hkl) 0° in black, those of (hkl) 90° in red. This expediency is really necessary for the study of the many, often intricate forms of quadratic crystals; besides such traces are in many respects more useful than the axonometric drawing of the crystal form.

The great number of distinct quadratic forms compels us here to give but the most important ones; in a separate memoir a thorough treatise of these forms will soon be given.

81. The holohedral holosymmetrical *dioctaid* (48) or *Dipyramid* of 16 faces is (Fig. 174)†

$$\omega\,[hkl] = \omega\,(hkl)\,(0^\circ + 90^\circ).$$

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*Sitzungsberichte, Wien, 1870, Bd. 62, I. Abth., page 9 of the memoir; also p. 13,

†We add in brackets, the *figure* in *Von Lang*, *Krystallographie*, and adopt the nomenclature given in that most excellent work. This will make the identification of forms easy without special figures here added.

The *diprism* (Fig. 175) ω [hk∞] of eight faces, and the *pinacoid* ω [∞ ∞ l] of two faces constitute the limits of this form in regard to l.

The *protopyramid* ω [hhl] (Fig. 176) and the *deuteropyramid* ω [h∞ l] (Fig. 177) (Fig. 178) and the *deutroprism* ω [$l\infty\bar{\infty}$] (Fig. 179.)

These 7 forms are holoquadratic holohedral, and therefore marked ω.

Of hemihedral holoquadratic forms we mention :

the *trapezohedron*, σ [hkl]. (Fig. 180).

the *deuterodisphenoid*, ζ [hkl]. (Fig. 183).

Hemiquadratic forms are also very numerous; here may be mentioned :

the *tritopyramid*, δ [hkl]. (Fig. 185).

the *tritosphenoid*, $\zeta\ \delta$ [hkl]. (Figure 191.)

Compare 57 to 60 inclusive.

82. IDOCRASE or VESUVIAN. *Von Kokscharow*, I., Pl. 10, 11. A masterly monography is that of *Von Zepharowich* on the crystal forms of this mineral, See Sitzungsberichte, Wien, 1864, 49, I., pp. 6—134, with 13 plates. Observed 46 forms of 488 faces.

Another example of quadratic crystals :

ZIRCON, *Von Kokscharow*, III., 139, 193; IV., 35; V. 103; Pl. 48, 49, 50, 51.

83. *Ammoniated Silver Sulphate.* See *Mitscherlich* in Rammelsberg Kryst. Chemie. Four forms of 18 faces.

Isomorphous therewith the corresponding selenate and chromate; also the ammoniated platinum sulphate.

Nickel sulphate, crystallized at about 20°, forms readily large and most beautiful quadratic crystals. See Rammelsberg, Kryst. Chem.

V.—Hexagonal Crystals.

*Elements** : $\alpha = \beta = \gamma = 90°$. $a = b\sqrt{3} \gtrless c$.

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*For the same absolute length of the vertical axis c (*Hauptaxe*) we may have our deutroaxis b either equal to the *Nebenaxe* a or to the *Zwischenaxe* s of *Quenstedt* and *Hochstetter*. Accordingly our proto-axis a will be either b = 2 s or 3-2 a of these authors. For the sake of brevity we shall term the first the system of A, the second the system of S. The latter is the most common, and gives the fundamental [∞11].

84. The octaid (46) in its primary position 0° and after rotation of 120° and 240° gives as resulting form, the holohexagonal holohedral holosymmetrical form:

$$\omega\,[hkl] = \omega\,(hkl)\ (0^\circ + 120^\circ + 240^\circ)$$

as the fundamental simple form of these crystals. It is the *dipyramid* (Fig· 131, *von Lang*, see note to 81) of 24 faces. See 49. All other hexagonal forms result, of course, from this form. Their generation and structure must be studied by the traces, as directed in 80; the third trace for 240°, may conveniently be drawn in blue, if the second (120°) is in red, and the first (0°) in black. Compare 57 to 60.

The number of forms in hexagonal crystals is still greater than in quadratic forms. Hence only a few will be given here. For a more exhaustive deduction of the hexagonal forms see notice at close of 80.

85. HEXAGONAL FORMS proper, or holohexagonal forms are:

HOLOHEDRAL
Dipyramid, ω [hkl]. Fig.* 131.
Diprism, ω [hk∞]. Fig. 132·
Protopyramid, ω [∞kl]. Fig. 133.
Deuteropyramid, ω [hhl.] Fig. 135.
Protoprism, ω [∞l∞].
Deuteroprism, ω [ll∞].
Pinacoid, ω [∞∞l].

HEMIHEDRAL
Ditrapezohedron, σ [hkl]. Fig. 137.
Protosphenoid Pyramid, γ [hkl]. Fig. 138.
Deuterosphenoid Pyramid, ξ[hkl]. Fg. 142.

86. RHOMBOHEDRAL FORMS, or protohemihexagonal forms are:

HOLOHEDRAL
Scalenohedron. π [hkl]. Fig. 146.
Rhombohedron, π [∞kl]. Fig. 148.

HEMIHEDRAL—*Trapezohedron* $\xi\ \pi$ [hkl]. Fig. 152.

87. GYROIDAL FORMS, or deuterohemihexagonal forms are:

HOLOHEDRAL: *Tritopyramid*, δ (hkl). Fig. 156.

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*See Von Lang's Krystallographie.

HEMIHEDRAL: *Tritohemipyramid* η δ [hkl]. Fig. 162.

88. Lateral hemimorphism (ξ or η, see 59) is not more common in hexagonal forms than terminal hemimorphism; (ζ in Turmaline;) hitherto, however, the former, have been overlooked.

89. BERYL, *Holohexagonal*, *Von Kokscharow*, II., 356; III., 72; IV., 125, 258. Pl. 12, 13, 14, 15, 16. Observed 18 forms of 266 faces.

90. PHENACITE; *Rhombohedral.* *Von Kokscharow*, II., 308; III., 81; V., 329; Pl. 39, 40, 41, 42. Observed 12 forms of 80 faces in all.

Another excellent rhombohedral example is:

CALCITE. *Hochstetter*, Denkschriften, Wien, 1852, Bd. VI., pp. 89—124, with a splendid large projection.

91. MARIGNACOIDS, the Chloroid–Salts $\pi\ a_3\ \nu$ wherein a any chloroid Fl, Cl, Br, Io, to which are *added* n atoms of water (n=1, 2, 3 observed) and

π = Mg, Zn, Cd; Mn, Fe, Ni, Co; Cu; Nd.
= Pt, Sn, Si, Zr.

See also the observations of *Topsoe*, Oversigt Kgl. D. Vid. Selsk. Forh. 1868, p. 123—157, and 1869, p. 73—97.

VI. A.—Quadro-Tesseral Crystals.

Elements: $\alpha = \beta = \gamma = 90°$. a = b = c.

92. The *Adamantoid* or *Hexakisoctahedron* ω [hkl] results as shown in 50 by taking successively each of the three equal axes as quadratic axis for a rotation of 90° of the holo-rhombic octaid ω (hkl).

If, however, we *omit* this rotation, but only turn the vertical l of the octaid successively into the three axes, we obtain the hemihedral form of twenty-four faces,

$$\frac{\omega}{2}\,[hkl] = \omega\,(hkl)\,0° + \omega\,(hlk)\,0° + \omega\,(lhk)\,0°.$$

called the *Diploid* by *Von Haidinger*; also known as the *Diakisdodecahedron*.

This form is the true hemihedral form of tesseral crystals. No other crystals admit of this kind of hemihedry. Hence this form is properly denoted by $\frac{\omega}{2}$.

93. If instead of the octaid ω (hkl) (see 57) we rotate the sphenoid σ (hkl) (see (58) we obtain the

σ [hkl] *Sphenadamantoid.—Haidinger.*
(Hexakistetrahedron.)—*Borazitoid.*
$\frac{\sigma}{2}$ [hkl] *Sphenodiploid.—Tetartoid.*
(Diakishexahedron.)

94. If we rotate the orthozone δ (see 59) we obtain δ [hkl], the *Gyroid* of *Von Haidinger*, having 4. 2. 3 = 24 faces, which are equal, irregular pentagons. The $\frac{\delta}{2}$ [hkl] is a Tetartoid. But these gyroquadrotesseral forms have, as yet, not been actually observed.

The same is true for the forms π and the monopyramidal forms ξ ε α, see 59.

95. By the variation of the indices each of these four-general ω, $\frac{\omega}{2}$ σ $\frac{\sigma}{2}$ gives rise to a great multitude of specific forms. Of the latter those which contain exclusively the *limits* of the indices (1 or ∞) are especially important.

For h = k = l = 1 the holohedral forms *geometrically* coincide in the regular *octahedron* ω [111] = σ [111] and the hemihedral in the regular *tetrahedran* $\frac{\omega}{2}$ [lll] = $\frac{\sigma}{2}$[111]. But *crystallographically* the hemihedral forms have but *one half* as many faces; for in ω [lll] each face is the result of *six* coincident planes, but in $\frac{\omega}{2}$ [lll] only *three* faces coincide. This difference largely influences the resultant combinations.

For h = k = 1 and l = ∞ the regular *Dodecahedron* results, while h = 1, k = l = ∞ gives the regular *Hexahedron.*

It will readily be seen, that [111] is generated by the pyramidal faces 111 in the rhombic octaid, while the dodecahedron results from the domes 11∞ and the hexahedron from the pinacoids, 1∞ ∞.

96. These forms should be placed in the following order, so as to constitute a complete *system :*

111........Octahedron........111

w $\frac{w}{2}$

Dodecahedron........Hexahedron......Dodecahedron.

11∞ σ ∞1∞ $\frac{\sigma}{2}$ 11∞

111.......Tetrahedron.......111

All other forms take their place in this system between the respective limits according to the actual values of their indices. The whole system of forms thus is composed of four equilateral triangles, three of which have already been given in the classical *Handbuch der bestimmenden Mineralogie* of *Haidinger*, (pp. 88—89; p. 102 and p. 107.)

97. The following are the principal intermediate quadrotesseral forms with their composite names and the specific names proposed by *Von Haidinger*. In our formula we have always h<k. The first number indicates the Figure in *Von Lang's* Krystallographie:

100....Triakisoctahedron....w [hhk].........*Galenoid.*
99....Trapezohedron.......w [hkk].........*Leucitoid.*
101....Tetrakishexahedron ..w [hk∞].........*Fluoroid.*
106....Trigondodecahedron..$\frac{w}{2}$ [hkk].........*Kyproid.*
107....Triakistetrahedron..$\frac{w}{2}$ [hhk].......*Deltohedron.*
112....Pentagonaldadecahedron σ [hk∞]..*Pyritohedron.*

Of each of these intermediate forms several have been observed. The same is true for the forms [hkl] see 92 and 93. The most common are those wherein h, k and l are 1, 2, 3, 4, 5 and 6.

On most of these forms we can readily distinguish the octahedral edge o, the dodecahedral d and the hexahedral h. For these edges the *internormal* angles have been calculated and may be given in a table.

Quadrotesseral Forms observed.

98. MAGNETITE.—*Von Kokscharow*, III., 47, Plates 45, 46.

This species is undoubtedly *quadrotesseral.* Observed forms all holohedral, viz:

c	1∞∞	t	133
d	11∞	z	2–5–15 or 3–5–3.5
o	111	x	5–15–21 or 5–3.5–3.7

Total 146 faces.

All the members of the great *spinel-group* crystallize in quodro-tesseral forms. Also galenite, halite, etc.

To this type belong also many artificial crystals, especially many chloroid-salts.

The grand group of *alums* (Physics, 194) form beautiful quadro-tesseral crystals, the formula of which is given by me in Sitzungsberichte, Wien, 1870, I Bd., p.

VI. B.—Rhombotesseral Forms.

Elements: $\alpha = \beta = \gamma = 90$. $a = b\sqrt{3}$.

S: $c = b\sqrt{2}$, if b = Quenstedt* s; cube ∞11;

or A: $c = b\sqrt{2}$, if b = Quenstedt a; cube 111.

99. For the general features of these forms, see 51.

The forms of the tesseral system may be expressed as rhombotesseral by reducing to our own the symbols given by *Hochstetter*, Denkschriften, Wien, VI., p. 116—121.

100. Mathematically the rhombotesseral forms (99) do not differ from the quadrotesseral forms described before. But physically and chemically, and therefore crystallographically, the rhombotesseral forms are wholly distinct from the quadrotesseral forms.

Thus Fluorite and Halite are both tesseral. But

$$\text{Fluorite is}\ldots Ca\left\{\begin{matrix}Fl\\Fl\end{matrix}\right.$$

while Halite is $Na\,Cl$; or chemically the atoms of the former are composed of three atoms, the latter of two only. Physically the latter has a cubical cleavage—yielding rectangular blocks, while the latter has the rhombohedral and basical cleavage resulting in octahedral fragments.

Pyrite $Fe\,S_2$ is also rhombotesseral; its hemihedral form is in this position *trapezoidal*, precisely as we have it for the rhombohedral quartz, $Si\,O_2$.

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*Compare foot-note to 84.

From among artificial crystals the nitrates of lead, barium and strontium are rhombo-tesseral; their formula is $R X_2$ where $X = O_3 N$, hence their chemical constitution the same as that of pyrite; they also exhibit the trapezoidal faces.

Garnet must also be considered a rhombo-tesseral species; its formula is $R_2 X_3$ wherein $X = R O_4 Si$. For its form, see *Von Kokscharow*, III., 1, 79, 230, and Plate 44.

The numerous so-called double chlorides, which are really *chloro-salts*, crystallize tesseral if they contain six atoms of the chloroid, and are undoubtedly rhombo-tesseral.

In many cases it is, however, at present impossible to distinguish between the rhombo-tesseral and quadro-tesseral forms.

CHAPTER V.

Crystal Symmetry.

101. In *abstract geometry* one polyhedron is as possible, as another. Hence the elements a, b, c, α, β, γ of such polyhedron can have *any* value, and the indices h, k, l may also be any numbers, rational or irrational.

The act of crystallization of any substance represents therefore, geometrically a double selection. First, *specific*, the elements assuming only definite values from among all possible values; and, secondly, from among all geometrically possible faces only those are selected which make the indices *rational*, *entire*, and *simple* numbers, giving the different *varieties* of the same specific form.

102. From this double geometrical selection it follows: I. that the *elements of a crystal* represent the inclination (α, β, γ) of the three systems of parallel lines along which the atoms of the substance are distributed and the *mutual* *distances* a, b, c of these particles along these lines;[*] and, II., that the indices h, k, l represent the *rates of growth* along these three directions. The first characterizes the *species*, the second the *variety* of the crystal forms.

103. If so important results concerning the molecular structure of solids depend upon the crystallographic elements and indices, the reality of these elements and indices should be most critically established.

We shall in the subsequent only give the outline of such a research.

104. The number of varieties of forms actually observed for many substances is quite large. The number of faces observed on Pyroxene is 176, Topaz 184, Beryl 266, and Idocrase 488. Now all these faces are for any one substance *accurately* expressed by the one set of elements and the simple indices given in last chapter (see 75, 78, 89 and 82.)

105. The substance richest in varieties of crystal forms is the rhombohedral *calcite.* In *Hauy's* atlas (1801) these forms occupy already four plates in quarto. *Bournon* Traite complete de la chaux carbonate fills 3 volumes (1808.) *Levy* gave 178 figures of calcite crystals (1837.) *Zippe's* Übersicht der Krystallgestalten des rhomboedrischen Kalk-Haloids[†] of 1851, enumerates 700 varieties of habitus, and of simple forms, 42 rhombohedra, 85 scalenohedra, 7 dihexahedra and several prisms, or about *fifteen hundred distinct faces!* These were united in one large projection by *Hochstetter*, 1854. Since then *Hessenberg* has discovered many new forms, yet *G. vom Rath*[‡] considers the description of the forms of calcite far from complete, although he himself at the same time richly adds to these forms!

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[*] The atomic structure of quadrotesseral and rhombotesseral crystals is therefore radically different. Compare 100.

[†] Wiener Denkschriften, Bd. III.

[‡] Pogg. Ann. 1867, Bd. 132, p. 387—404 and 517—550, with 1 plate. Also Pogg. Ann. 1868, Bd. 185, p. 572—579. He adds, so far as I can enumerate, 5 new rhombohedra, 5 scalenohedra, and 2 dihexahedra, or more than *one hundred new faces!*

Now it is a *fact*, that all these faces are expressed accurately by the *same* elements and by simple rational indices!

106. The indices are, in the immense majority of cases, not only rational whole numbers, but also *very small whole numbers.* Compare the descriptions given, Chapter III. However, in a few rare cases the indices become rather high; but then the corresponding faces only occur as narrow *transition belts* between faces of very simple indices. These high indices result furthermore from the simple indices hkl of the dominant form by multiplication with a usually large factor *f* and the addition of some small numbers to one or two of the resulting products; so that the *vicinal face*† resulting has indices like

$$f.h+p : f.k : f.l+p.$$

107. *Von Kokscharow* observed the vicinal face 61, 20, 61, or 3.20+1 : 1.20 : 3.20+1, near s = 313 on Vesuvian. See also *Von Zepharowich*, Idocrase, p. 127—128, who unites the four faces n, w, m, k of *von Kokscharow* in one pyramid. Compare 82. Also, *Kenngott* Übersicht 1862—1865, p. 182—183.

On Fluorite von Kokscharow (V. 200) observed the adamantoid $_{\omega}$[15-33-55] where the indices are composed of the prime factors 3, 5 and 11. This is most frequently the case where any one index, for example c, is much greater than the others; thus in 236 we have l.= h. k.

108. The elements are, for any given substance, really constant. But they vary with the temperature, as is shown in any work on crystallophysics. Compare Contrib. III., p. 43—45.

If the substance is not chemically pure, the elements of its form will of course be influenced by these accessory ingredients. For examples see *Auerbach* on celestine (quoted in 69) and *von Kokscharow* on apatite (V. 86–100) Compare 55; also 91, 79, 76.

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† *Websky's* term, which we adopt, without, however, adopting the peculiar views of *Websky*.

109. The correctness of these elements implies certain mathematical conditions for the forms of higher symmetry,)see 46 to 51.) Thus quadratic forms must be *exactly* quadratic; hexagonal forms must be *exactly* hexagonal. On this point we have very accurate measurements, especially by *von Kokscharow* on Beryl (89) and by *von Zepharowich* on Idocrase (82). Most crystals, it is true, do not possess the perfect, mathematical regularity of form required by the above conditions.

110. *Von Zepharowich* found, however, *one crystal* among 54 crystals of Idocrase from the Mussa-Alpe in Piedmont, which had its four pyramidal faces p = 111 inclined under the same angle of 142° 45′ 0″ (internormal, 37° 15′ 0″) to the base c = ∞∞1. See l. c. p. 65, I.

But the deviations observed on the other crystals prove by their very minuteness, and by their variations, that they are due to accidental causes, such as slight impurities, position during growth, pressure, etc. Thus No. 11, Fig. 24, of *von Zepharowich* (l. c. pages 62–63), a grass-green crystal, 11 mm high, 5 and 6 mm across, gave for the internormal angle between any two adjacent faces p values ranging from 50 40′ to 50° 41′. But a variation of one minute on a crystal of this size is less than the variation of one-thousandth of a millimeter in the linear dimensions!

111. *Von Kokscharow* made similar careful measurements on two fine crystals of Beryl. He found the angle of the prism truly 120° 0′ 0″, by repeated measurements (compare 89), and the interfacial angle between the six pyramidal faces t and the base P varied only about 20 seconds from 160° 3′ 42″, the mean value for his crystal No. 2. Since that crystal had a diameter of about 5 mm only, and as the linear extend of the facets t was less than one-fifth the diameter, this deviation amounts to less *than* *one ten-thousandth part of one millimeter*, for the linear position of t on the lateral edges of the crystal.

112. Even the crystals of *diamond*, which as a rule have curved faces, exhibit at times almost mathematically perfect crystals. The colorless diamond described by *von Kokscharow* (V. 397) was such an octahedron; in one zone the deviations from the normal interfacial angles 109° 28 16″, and 70° 31′ 44″ were only — 14, — 44, — 46, — 16 seconds.

113. Crystals do therefore really exhibit that two-fold geometrical selection (101) whereby they are distinguished as true individuals from all other mere polyhedra.

Artificial crystals are often of most surprizing regularity, not only in angles but also in their linear dimensions. By changing the conditions of crystallization we can produce quite regular or more distorted crystals. See the careful observations and the description of most beautiful crystallizations by *Karl von Hauer*, Wien, Sitzungsberichte, 1860, Bd. 39, pp. 438–447, and pp. 611–622; also Bd. 40, pp. 539–554, with 2 plates, and pp. 589–606 with one plate.

114. The degrees of symmetry described are merely *limits* of form, whereto many neighboring forms approach, or *transition forms*, having allied forms on either side. Compare 55 and 56.

This is not only true in regard to the axial dimensions, but also in regard to the *correlate faces* embraced in one simple form in the higher forms of symmetry.

115. Thus in *triclinic forms* we have very frequently simple forms which correspond to a monoclinic simple form, and would therefore form such, if only the elements were correspondingly changed. Compare in Anorthite (72) a and m; p and o; these four together would even form the orthoclinic octaid. Also observe how near β and γ are to 90°.

Furthermore we have the monoclinic combinations b and β; w, v; g, u; etc., etc. Also T, l; z, f. Compare *Haidinger*, Handbuch, p. 223, Fig. 336, and his remarks thereon. Also, *Tschermak's* genial work on the *Feldspars*, Wien, Sitzungsberichte, 1864, Bd. 50, I., p. 566–613.

117. *Rhombic crystals* possess often all faces demanded by quadratic symmetry, if the axis a is nearly equal to b. For example in *epsomite* and in white vitriol. Observe the horizontal projection of epsomite in *Rammelsberg's* Krystall. Chemie, 1855, p. 87. This near approach to higher symmetry is also manifested by the frequency of the sphenoid in this species.

118. Rhombic crystals, wherein a is nearly b $\sqrt{3}$, have all the forms characteristic of *hexagonal* symmetry. This is strikingly the case with all the members of the isomorphous group described in 79. The hexagonal prism is p and b; pyramid o and q^2, another $^o/_2$ anb q.

In phenacite (90), a *member of this group*, the axis a is *exactly* equal to b $\sqrt{3}$, and here these forms therefore not merely represent or simulate, but really *do constitute* a true hexagonal symmetry!

In Topaz a — b $\sqrt{3} = 0.160$, a rather large difference. Still we have the planes necessary for many hexagonal forms.

Compare also *Discrasite*, the prismatic angle of which is 119° 59′.

119. *Quadratic crystals* have most of the characteristic faces of quadro-tesseral crystals, if the vertical axis c is nearly equal to the horizontal axis. Thus in the quadratic *chalcopyrite* $c = 0.98556 = a - 0.01444$ where $a = b = 1.00000$; hence we find the habitus of chalcopyrite totally quadrotesseral, even in its hemihedral forms. This simulation is so great, that the form of chalcopyrite was considered really quadro-tesseral until *Haidinger* discovered its true quadratic form.

Idocrase (see 82) gives $2\ c = a + 0.07508$; hence it has also a quodro-tesseral symmetry, as shown by *Haidinger*, Handbuch, p. 214.

120. *Hexagonal crystals* have nearly a rhombotesseral symmetry, if their vertical axis c is nearly $b \surd 2 = 1.414$ b or $b' \surd {}^{3}/_{2} = 1.215\ b'$. Cases are quite frequent where the rhombohedron is either near 90° or near 120°.

121. But besides the above relations between the different degrees of symmetry we have many others. In fact, any degree of symmetry may and does approach any other very closely if the elements differ but little *numerically.* Hence the first observers assume often a higher degree of symmetry than more accurate observations can sustain (compare 119), while, inversely, distorted crystals may for some time hide a higher degree of symmetry.

The trihydrated potassium cyanoferrate* has by *Bunsen* and *Rammelsberg* been described as quadratic; but *Wyrouboff*† found it to be monoclinic, having the inclination $\alpha = 89° 27$ instead of 90°, while $a = 0.395$, $b = 1.000$, $c = 0.401$. The differences between the real and quadratic elements are only 33 minutes for the interaxial angle α and $0.006 = c - a$.

Tinfluoride, $Sn\ Fl_2$ is monoclinic (*Marignac*); but very nearly tesseral.

Cryolite is triclinic (*Websky*, 1867); but its three pinacoids differ only 24, 10 and 1 minute from being at right angles to one another, and the axes are also nearly equal. Hence it is very nearly tesseral.

Such cases become quite numerous, when the crystal elements are critically compared and not merely mechanically arranged in formidable tables.

122. From this it appears that the classification of substances according to their crystallographic symmetry (or system) iz a crystallographic absurdity; and every attempt of the kind has served to prove that such a classification leads to physical impossibilities and chemical monstrosities. Compare Atomechanik, § 352—254.

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*Yellow prussiate of potassa.

†Annales de Chimie et de Physique, 1869, T. 16, p. 293—301.

123. By overlooking these connections between the various crystal forms, the different degrees of symmetry were considered *qualitatively* radically different, while in fact they are only *quantitatively*, and often to but a very slight degree different from one another. Hence some crystallographers have been led to the monstrous invention called dimorphism. Compare *Atomechanik* pp. 31 to 41; also Contributions to molecular science, I. p. 9—15, and III.

124. To this same kind of physical monstrosities belongs the invention of *P. Groth*, termed by him *Morphotropic force* and morphotropy. This invention was presented to the Berlin Academy of Sciences by *G. Rose*, and has, therefore, been copied by several scientific journals. Compare 163.

125. Such monstrosities as the common view of dimorphism and this new morphotropy are rooted in the current perversion of *Mitscherlich's* discovery of isomorphism. He found that the salts of a certain chemical formula are of nearly the same form, and called this fact *isomorphism;* but his successors have succeeded to turn this fact upside down, and they now conclude that the vicariating atoms are of the same form.[*] Compare 159 and 156.

126. We sincerely regret to be compelled to this forcible protest against the continued disregard of the most elementary principles of mechanics by some of the leading crystallographers, especially of the Berlin school. Their monstrous speculations have, however, so deeply permeated the crystallographic literature that no protest can be too strong.

127. The introduction of *rotary symmetry* for the deduction of the quadratic, hexagonal and tesseral forms

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[†] *Rammelscerg* in Ber. Chem. Ges., 1870, p. 281, discovers (!) that hydrogen may be an isomorphous component in the phosphates $R'_3 O_4 P$, because one or two atoms R' may be hydrogen. The form of this phosphate is not determined by the atomic structure — but by hydrogen!

from the rhombic forms implies a twin-structure for these crystals of highest symmetry; but *a twin-structure composed of elements which in the several positions are of precisely the same dimensions.*

128. Such structure has been demonstrated by observation in several cases already; especially in *Quartz*, by *von Haidinger*, in the remarkable paper Über den Pleochroismus und die Krystallstructur des Amethystes, Sitzungsberichte, Wien, 1854, Bd. 12, p. 401. Also by optical observations of *Descloizeaux.*

These observations have been referred to the chemical formula and form of the quartz-atom by me in Atomechanik, §497—499, and in Über den Bau des Quarzes, Sitzungsberichte, Wien, 1870, Bd. 61, I. Abth.

129. But even if a crystal is not exactly hexagonal, but approximates to the axial ratio $1 : \sqrt{3}$, such twin-structure becomes quite frequent. In *Haidinger's* memoir* "on the regular composition of crystallized bodies," I find the earliest thorough investigation of this structure.

The figure of cerusite, PbO_3C given by *Haidinger*, especially Figures 6, 7, 8 and 9, exhibit precisely the structure which for the axes $a : b = \sqrt{3} : 1$ would lead to true hexagonal forms. The same forms are frequent for the isomorphous anhydrous carbonates and other crystals of nearly hexagonal axes.

130. The beautiful crystals of *chrysoberyl*, and especially the *alexandrite* variety, exhibit this twin-structure often on a grand scale, forming most beautiful specimens. See *von Kokscharow*, Atlas, plates 63, 64. The axes are very nearly in the above ratio, because the angle of the vertical prism is 119° 46′, or only 14 minutes less than 120° 0′.

131. Twins demonstrating the quadrotesseral structure given by us have also been described by *von Haidinger ;*

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*Edinburgh Journal of Science, VI., 278. This paper I recently received from the author, whom I hereby most heartily thank for this and many kindred favors.

Fig. 423, p. 267 in Bestimmende Mineralogie. This crystal of chalcopyrite is a complex of six individuals, grown together so that indeed each extremity of the three rectangular axes forms the axis a of a quadratic crystal. This twin is so closely like a true tesseral, because the vertical axis of chalcopyrite is so nearly equal to the horizontal axes. See 119.

132. All forms are geometrically equally probable (101); and since the number of possible distinct forms is much the greatest for those lower degrees of symmetry due to unequal values of the elements, the less symmetric crystals should, geometrically be the most common.

133. If we suppose the axes determined to within 0.001 and the interaxial angles to within one minute, the relative *geometrical probability* of the different degrees of symmetry will be* for every t tesseral forms,

tesseral = t
hexagonal, h = 500. t
quadratic, q = 1000. t
rhombic, p = 1 000 000. t
monoclinic, m = 500 000 000. t
triclinic, cl = 250 000 000 000. t

134. But by a combination of these values with the actually observed number of each degree of symmetry we obtained (l. c. page 15) the *true crystallographic probability* of

tesseral symmetry	800 000 000 000
hexagonal "	820 000 000
quadratic "	274 000 000
rhombic "	1 000 000
monoclinic "	2 000
triclinic "	1

135. It is therefore 800 000 000 000 to 1 that a new crystal has a tesseral rather than any *given* triclinic form,

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**Hinrichs*, Zur Statistik der Krystall-Symmetrise. Sitzungsb., Wien, 1870, Bd. 62, I. Abthl., p. 14.

and 800 000 to 1 that it has the tesseral rather than any given rhombic form.

136. The greater the number of conditions to be fulfilled to bring about a given form of symmetry, the more likely will that form be produced by the act of crystallization.

137. These remarkable facts apparently constitute a crystallographic paradox, thus indicating some general cause which mathematically accounts for these facts.

138. We have already in 1867 shown that the laws of chemical or atomic combination lead with mechanical necessity to an atomic structure of compounds which in the crystal-form is expressed by masses. See:

Programme der *Atomechanik*, oder *die Chemie eine Mechanik der Panatome*, von Gustav Hinrichs; Iowa City, Vereinigte Staaten, N. A., 1867.—44p., 4o.

139. Since then six memoirs have been published on special points of this programme; we shall, for the sake of brevity, refer to them as *Contributions to Molecular Science*, No. 1 to 6. Of these Nos. I. to IV. appeared in the *Proceedings of the American Association* for the Advancement of Science, Vols. 17 and 18 (1868 and 1869); they may, however, be obtained separately from *Brockhaus*, Leipzig. Nos. V. and VI. are contained in *Sitzungsberichte*, Wien, 1870, I. Abtheilung; namely

V. *Ueber den Bau des Quarzes*, in Vol. 61.

VI. *Zur Statistik der Krystall Symmetrie*, in Vol. 62.

Thus *Contrib. VI.* refers to this last mentioned paper.

140. In these "*Contributions*" and in "*Atomechanik*" the crystal form of a great many series of compounds has already been theoretically determined. We shall, in the next chapter, give a synopsis of this investigation, with references to the above works.

This synopsis is only to be considered a temporary substitute for a more extended work which has been a long time in preparation, but which official duties have not yet permitted me to bring to a satisfactory conclusion.

CHAPTER VI.

Crystal Atoms.

141. All chemical compounds which can be qualitatively and quantitatively expressed by the same *general formula*, constitute a *type*. Atomech. §232. Contributions, VI., p. 9—11.

142. In a *general formula* the symbols represent no given individual elements but either certain definite chemical compounds (Contr. VI., p. 9, No. 22) or *any one* individual element from a greater or smaller group of elements, (l. c. p. 7).

143. The elements are divided into *genera* (Atomech. 19 to 37, pp. 7—9, and Contr. IV., p. 48—51; also Contr. VI., p. 7), each of which is represented by a generic *Greek symbol.*

144. The elements are also distinguished as electropositive π and electronegative ν, together with intermediate α in ternaries. Contr. VI., p. 6.

145. The simple types therefore are only *specific functions* of π ν, and (if ternaries) α. If they are *hydrated* the atoms of water may often be combined with α and this combination denoted by ω. The same may be done with certain ammoniacal compounds. (Compare 83 above.)

146. Accordingly the general chemical formula

$$\psi\ (\alpha,\ \omega,\ \gamma)$$

does represent the great majority of the more important types as some definite function ψ of α, ω and ν.

147. The atom ψ has its constituent atoms in some geometrically well defined position in space. Let a, b, c represent the three axes of this compound atom, and α, β, γ the interaxial angles of the same.

148. Then those *atomic elements* are necessarily definite functions of ψ.

This can only be denied by those who dream of occult forces (124.)

149. *The interatomic distances* (between the compound atoms) are *proportional* to a, b, c; compare 102; also Atomech. Abschnitt IV. *Hence the atomic elements are the same as the crystal elements.*

150. Let, for b = 1, the interatomic distance along b be in some absolute measure. Then the crystal volume of the atom is easily calculated; and by comparison with the observed atomic volume the *mean atomic distance* μ is obtained.

151. For many orthoclinic crystals we have determined μ and obtained very remarkable results. This paper was presented August, 1869, at Salem, but not read for want of time. Proceedings, p. 275, No. 3.

152. The determination of the atomic elements as functions of 146 is so intricate an analytical problem, that it can only be solved by successive approximations. Atomech. §229—231.

153. As *first approximation*, we neglect the differences in weight between the constituent atoms; the resulting values of the elements we term the *normal elements of the type* 146. Atomech. 231; Contrib. 3, p. 36. The normal axes we call x, y, z.

154. The *actual values* (149) then result mechanically from the *perturbations* of the normal elements, exactly as the actual elements of any planet result from the perturbations of the other planets affecting its normal elements, determined by the sun alone.

155. Let these *perturbations* be ξ, η, ζ, then we have

$$a = x + \xi \qquad b = y + \eta \qquad c = z + \zeta$$

Compare Atomech. 233; Contrib. 4, p. 36. Similarly for the interaxial angles.

156. Hence it follows:

1. *The different members of the same chemical type have nearly the same crystal elements;*

2. *But these elements may belong to all degrees of symmetry according to the numerical values of the linear and angular perturbations, resulting from* 146.

157. For a general confimation hereof see Contrib. VI., pp. 9—11.

In this table the degree of symmetry of the observed n members of any type or genus is expressed by the formula

$$n \ldots\ldots\ldots\ldots\ldots q.\ t.\ h.\frac{p}{m.\ tr}\ldots\ldots\ldots R.\frac{p}{cl}$$

wherein *q* the number of quadratic members, *t* the tesseral, *h* the hexagonal, *p* the prismatic (or rhombic), *m* the monoclinic and *tr* the triclinic members. Also, R the sum of g't and h, or the total number of members having rotary symmetry, and *cl* = m and tr the clinogonal species.

158. Thus the *Barytoid Sulphates*, that is the normal sulphates (79) including three observed selenates, give the following numerical values:

$$15 \ldots\ldots\ldots\ldots 0.\ 0.\ 2.\frac{13}{0.\ 0}\ldots\ldots\ldots 2.\frac{13}{0}$$

ranging over two different degrees of symmetry the rhombic and hexagonal; but the angle of the rhombic prism is nearly 120, so that these forms are quantitatively very close.

159. The above principle (156) represents true *isomorphism* or homoeomorphism; what is usually taught under isomorphism is false in fact and absurd in principle*. Compare 125.

160. In conclusion we shall enumerate the types which we have thus far more carefully determined by theory (15—155) and confirmed by existant observations.

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*The relation between the number of atoms of the negative elements and the degree of symmetry put forth in 1867 as original by *J. D. Dana*, the editor in chief of the American Journal of Science, resulted evidently from hasty and illogical study of my manuscript on the crystalline form of anhydrous carbonates, which manuscript with figures was in the hands of that editor several months prior to the publication of his "original discovery." See *Documents* relating to the history of Atomechanics, Iowa City, May 1868. Also, *Remarks* on a recent Editorial in the American Journal of Science, Iowa City, December, 1868.

161. The *Monaries* A or elements; see Atomech. §256—264.

162. The *Monides* $^{\pi}\nu$; see Atomech. §265.

163. The *Deutides* $_{\pi}\,\nu_{2}$; see Atomech. §§267—301. Also Contrib. I., p. 5, pp. 9—15. III., pp. 37—38, and p. 104. Contr. V. Contr. VI., p. 9. We sent December, 1868, a paper, giving a close discussion of 53 compounds of this type to *G. Rose.*

164. The *Tritates* $^{\pi}{}_{\omega 3}\,\nu$. Atomech. §§. 302—356. Contrib. III., pp. 38—41, also pp. 44—45, and VI., pp. 9—11.

165. The *Tetrates* $_{\pi\;\omega 4}\,\nu$; see Atomech. §. 361—369 for spinnelloids; my "Note sur la forme cristalline des sulfates" mentioned in 79 for the barytoids. Also Contrib. III., p. 41—45, VI., p. 13.

166. Other more intricate types have already been partially determined. Atomech. §§. 370—379; also here 89, because 2 c as vertical brings beryl to the form Atomech. §. 271. Topaz has also been accurately determined from its chemical formula given in 78.

167. A more detailed explanation of the formation of snow-crystals we gave in the Scientific American, New York, April 18, 1868, with 7 figures.

168. The relation between physical properties and molecular crystalline constitution has been treated of in Atomechanik, p. 42 – 44, Contrib. III., p. 43—46, and in Contrib. V. Also, in a paper of 1868, "on the reality of rhombo-tesseral forms," which has not yet been published.

169. The above must be sufficient to show that the problem (152) can be solved, because it *has* been solved in several important cases.

170. In conclusion, we express the hope that, at no distant day, we may be enabled to present the subject of this chapter in a more complete form, as a revised and largely improved edition of the fourth section of *Atomechanics*, constituting a treatise of crystallophysics and crystallochemistry, referred to in the preface.

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